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Developmental dynamics between reading and math in elementary school

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Abstract

Reading and math attainment develop during elementary grades. Questions remain, though, about the co-developmental nature of the relation between reading and math. This study examined dynamic, longitudinal pathways between reading and math in first through fourth grades. Participants of the study were 554 academically at-risk children (M_{age} at the first assessment point = 6.57 years; SD = 0.38) from Texas Project Achieve. Children were assessed utilizing the Woodcock-Johnson–III reading and math measures. Results from dynamic bivariate latent change score models indicated unidirectional longitudinal coupling effects from reading to math. Specifically, average and high levels of reading performance were associated with subsequent gains in math growth, in particular for below average performing children in math. In contrast, low levels of reading performance had negligible or no amplifying influences on change in math growth. The nature of the dynamics was replicated even when controlling for nonverbal cognitive abilities. Results demonstrated that good reading skills pave the way for children to develop their math skills. Such findings underscore the importance of considering reading performance in treating math difficulties.

KEYWORDS

academically at-risk, co-development, elementary grades, latent change score modeling, math, reading

1 | INTRODUCTION

Children experiencing difficulties in reading and math represents an important public health issue, as struggles with reading and math are associated with consequences regarding academic and life success, including but not limited to academic failure and lower socioeconomic status as an adult (Geary, Hoard, Nugent, & Bailey, 2012; Ritchie & Bates, 2013). Highlighting the scope of this problem, a large proportion of U.S. students struggle in reading and math. Startling enough, 20%–25% of fourth-graders fail to reach even partial mastery of grade-level knowledge in reading and math (National Assessment of Educational Progress, 2019). Given the reciprocal influences between reading and math (e.g., Cameron, Kim, Duncan, Becker, & McClelland, 2019) alongside the shared co-occurrence of reading and math difficulties (Landerl & Moll, 2010), it is noteworthy that their development has been mostly studied in isolation of each other (Vanbinst, van Bergen, Ghesquière, & De Smedt, 2020). This is surprising as knowledge on their potential mutual unfolding over time may help inform instruction and types of interventions in both academic outcomes. The aforementioned observation set the foundation for the present study to address the nature of the developmental dynamics between reading and math. The goal of the study was to examine the extent to which reading and math co-develop across elementary grades in academically at-risk children, precisely the key time period for the most rapid reading and math development. We utilized a state-of-the-art approach called latent change score modeling, which takes into account change processes in studying co-developmental questions.

1.1 | Theoretical framework underlying the developmental dynamics between reading and math

A study investigating the developmental dynamics between two constructs first requires an understanding of how each of the domains changes independently. Work to date examining reading and math development in a univariate framework has found rank-order stability in mean growth based on initial status of reading and math performance but also variability in the rate of growth (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Campbell, Pungello, Miller-Johnson, Burchinal, & Ramey, 2001; Foorman, Francis, Fletcher, Schatschneider, & Mehta, 1998; McCoach, O'Connell, Reis, & Levitt, 2006). Individual differences in the growth trajectories for both constructs have mostly indicated a fan-spread growth pattern, suggesting that children who are below average at the initial level grow more slowly than children who are above average (e.g., Aunola et al., 2004; Bailey, Watts, Littlefield, & Geary, 2014; Bast & Reitsma, 1998; Bodovski & Farkas, 2007; Pfost, Hattie, Dörfler, & Artelt, 2014). What remains lacking, however, is knowledge of simultaneous growth patterns and growth rate across reading and math. Additionally, it is unknown whether reading and math co-develop relatively independently or whether changes in one construct predict subsequent changes in the other.

Although there is scant research on the developmental dynamics between reading and math, clues regarding the direction of the developmental effects may be drawn from a line of research that studies reading and math in conjunction. One line of studies have found that reading and math are correlated, however with no direct time-dependent relations (e.g., Koponen, Aunola, Ahonen, & Nurmi, 2007; Koponen et al., 2019; Korpipää et al., 2017; Korpipää et al., 2019). This view is supported by the investment theory (Cattell, 1987). The investment theory posits that there is a general cognitive ability in young children that, through environmental stimuli such as education and experience brings about the development of academic performance. This 'investment' of cognitive ability into academic knowledge occurs extensively during the school years when the complex abilities needed to learn academics such as reading and math are mostly acquired (Cattell, 1987). According to the investment theory, reading and math are expected to be positively correlated because they have a common underlying cognitive factor. Empirical literature showing reading and math sharing common domain-general cognitive correlates, such as working memory, processing speed, rapid automatized naming, reasoning, and serial retrieval fluency is abundant (e.g., Fuchs, Geary, Fuchs, Compton, & Hamlett, 2016; Koponen et al., 2007; Koponen et al., 2019; Korpipää et al., 2017; Korpipää et al., 2019; Vanbinst et al., 2020).

The body of research, which suggests that the relation between reading and math is simply correlational, is counterbalanced by the other set of studies that predicts a directional association between

Research highlights

- Reading and math are related academic outcomes, yet the developmental dynamics between reading and math have not been studied extensively.
- We used a latent change score modeling approach to study the co-development of reading and math in academically at-risk elementary school children.
- Results supported a reading-to-math coupling model indicating that good reading was associated with gains in math growth, in particular for below average performing children in math.
- The findings lend support to the idea that early reading skills are associated with success in math.

reading and math. There are three possible directional relations, each of which has theoretical and empirical support. Firstly, studies have found that reading (related) skills (e.g. language and phonological processing as pre-reading skills) are predictive of math development. This option has found a place in some developmental mathematics theories, for example, the triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995) and the bootstrapping theory (Carey, 2004). The theories have in common the thought that language shapes the development of numbers concepts. Dehaene and colleagues (Dehaene, 1992; Dehaene & Cohen, 1995) have argued that the verbal code is particularly important at the beginning of formal math learning because it links the visual Arabic number code with the analogue magnitude representation code. Carey (2004) has taken this premise a step further and suggested that language even has a causal influence on at least some aspects of numeracy. The idea of causality, though, has been challenged by Gelman and Butterworth (2005) who showed that numerical concepts have an ontogenetic origin and a neural basis independent of language. One particular component of language directly related to the possibility of reading influencing subsequent math is phonological processing. In the pathways to mathematics theory, LeFevre et al. (2010) have demonstrated that both phonological processing and vocabulary as linguistic skills served as important avenues to form visual and verbal representations of the meaning of numbers. The authors have rested the argument on the fact that learning the rules of the number system is assumed to be similar to mastering any symbolic representational system (e.g. written language). LeFevre et al.'s (2010) model has provided a framework for many empirical studies showing that children use phonological processing and fluency reading abilities to support their learning of math (e.g., Fuchs et al., 2006; Hecht, Torgesen, Wagner, & Rashotte, 2001; Rinne, Ye, & Jordan, 2020; Simmons, Singleton, & Horne, 2008; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). A further direct time-dependent evidence for a direction from reading to math comes from a study that showed that third grade reading comprehension had a positive significant influence on change in math skills up to eighth grade, such that children who performed well in reading comprehension tended to improve in math more rapidly than children with lower reading comprehension scores (Grimm, 2008).

Converse to the option of reading predicting math development is the possibility that math (related) skills facilitate reading development. In a hypothesis by Koponen, Salmi, Eklund, and Aro (2013), the authors suggested that some aspects of counting skills, such as counting by 2 s, might be a key predictor of later reading. The authors believe that fluent and accurate counting is fundamental to the formation and retrieval of arbitrary visual-verbal associations in long-term memory, which are key to math as well as reading fluency. In fact, counting ability measured in kindergarten has been shown to be associated with reading performance even more strongly than more traditional, linguistic predictors of reading (Leppänen, 2006). The predictive effect of counting on reading has remained even after controlling for phonological awareness and partially for verbal short-term memory (Koponen et al., 2013). Other studies have also reported that math related skills, such as cardinal knowledge and numeral recognition predicted subsequent reading. For instance, kindergarteners who were able to recognize (complex) numerals scored higher on a productive letter knowledge test – a prerequisite for reading – than children who did not recognize as many numerals (Vanbinst et al., 2020). Similarly, sensitivity to the relative quantities of collections of objects and cardinal knowledge in preschoolers predicted their reading achievement in kindergarten (Chu, vanMarle, & Geary, 2016).

Lastly, there is also research that has failed to find inverse predictions one way or the other, but has instead alluded that the relation between reading and math might be conceptualized as bi-directional or reciprocal. In other words, reading might predict math development, and math might predict reading development. This assumption is supported by theories of reading and math development (e.g., the developmental phases of sight word reading by Ehri, 2008, the framework for reading comprehension by Perfetti (1995), and the children's mathematical development theory by Geary, 1994). These theories illustrate that developmental course in reading and math proceeds with similar developmental milestones across both domains. Children enter first grade with a core suite of basic competencies. In reading, those include an understanding that sounds are coded as letters. In math, the foundation of formal math is related to the understanding of numerical magnitude (Starkey, Spelke, & Gelman, 1990), the rules for counting (Gelman & Gallistel, 1986), and how the addition and subtraction increases or decreases quantity respectively (Levine, Jordan, & Huttenlocher, 1992). Next, a period of procedural strategies ensues across both domains – alphabetic principle in reading (Ehri, 2008) and learning the associations between visual, phonological, and semantic representations of numerals in math (Geary, 1994). Then, these procedural strategies are repeated over time in both domains. As such, children secure representations of words and arithmetic facts in their long-term memory which, in turn, increases their reading and math fluency. Finally, a phase with increased cognitive complexity follows in both domain. In reading, children start with reading comprehension by retrieving the meaning of words from long-term memory, drawing inferences, and relating text ideas to their knowledge base (Perfetti, 1995). In math, children learn how to make decisions for operation calculations and applied problems using retrieval and other strategies (Geary, 1994). Given the two domains appear to develop along similar paths, this might suggest their co-mutual beneficial influences. A recent study by Cameron et al. (2019) showed that when children applied quantitative competencies during the kindergarten year, they were likely to make more literacy advances 1 year later. The reverse prediction was also true. Emergent literacy skills, such as letter-word identification, predicted solving applied quantitative problems. Bi-directional effects have also been reported in larger longitudinal studies using nationally (U.S.) representative samples (e.g., Bailey, Oh, Farkas, Morgan, & Hillemeier, 2020; Duncan et al., 2007), albeit the effects were fairly small (Bailey et al., 2020).

Overall, theories on cognitive (e.g., Cattell, 1987), reading (e.g., Ehri, 2008; Perfetti, 1995), and math development (e.g., Dehaene, 1992; Geary, 1994; LeFevre et al., 2010) alongside the empirical findings from cross-sectional as well as longitudinal studies provide a clear statement on the importance of overlap and potential co-development between reading and math. These theories indicate that cognitive, reading, and math components work in concert rather than independently, suggesting that unfolding of reading and math development likely happens in conjunction. In addition, empirical support shows significant, albeit contrasting findings in terms of directionality of relations between reading and math. Altogether, this has implications for research such as the present study which is aimed at considering all relevant pathways in conjunction when modeling growth and change in growth processes in reading-math co-development.

1.2 | Developmental dynamics between reading and math in academically at-risk children

The available research has studied the overlap and dynamics between reading and math mostly in typically developing children. In our study, we report data on academically at-risk children – here defined as children scoring below the median on an early literacy test prior to first grade (see Participants section). However, very little work has been conducted thus far on the reading-math co-development in this population. Hence, in the section that follows we will overview some literature addressing the reading-math dynamics in academically at-risk children but also children with reading and/or math disabilities. The latter research has the capacity to provide probable clues regarding the reading-math co-development also for academically at-risk children.

As previously noted, reading and math show a high degree of overlap likely due to sharing of domain-general cognitive correlates (Cattell, 1987). When discussing academically at-risk children, a similar premise is described by the multiple deficit model (Pennington, 2006). The model states, in part, that the comorbidity of reading and math difficulties is attributed to common genetic, environmental, and cognitive risk factors, such as processing and working memory deficits and demands on problem solving (Daucourt, Erbeli, Little, Haughbrook, & Hart, 2020; Erbeli, Hart, & Taylor, 2019;

Swanson, Jerman, & Zheng, 2009). According to the multiple deficit model (Pennington, 2006), we can predict positive correlations between the initial performance as well as growth within and across reading and math in academically at-risk children.

Research on direct time-dependent relations between reading and math in academically at-risk children is even scarcer than correlational literature. In terms of reading predicting subsequent math, a longitudinal study by Jordan, Hanich, and Kaplan (2003) showed some evidence for this possibility. In early elementary school, children with math difficulties who were good readers progressed faster in math than children with comorbid reading and math difficulties. In contrast, children with reading disabilities who were good in math and children with comorbidities progressed at about the same rate in reading. These findings suggest that reading performance might have influenced growth in math, while the reverse direction of influence was not the case. Moreover, the study demonstrated that reading difficulties appeared to remain stable throughout elementary school, whereas math difficulties were ameliorated by reading competencies, again alluding to high levels of reading performance being beneficial for math growth. Overall, according to mathematics theories (e.g., Dehaene, 1992; LeFevre et al., 2010) positing that reliance on language and phonological representation might contribute to math performance, it is suspected that reading contributes to math also in academically at-risk children. Based on empirical evidence (e.g., Jordan et al., 2003), we can expect that this direction is particularly notable in children with high levels of reading performance who have well-developed phonological representations (a shared pre-requisite for skilled reading and math, e.g., LeFevre et al., 2010).

In terms of math predicting subsequent reading, we predict that this outcome is less likely to be observed in academically at-risk children. Recall that for typically developing children, Koponen and colleagues (Koponen et al., 2007, 2013) suggested that counting principles contribute to reading outcomes through the retrieval mechanisms of arbitrary associations between visual symbolic and phonological forms. Importantly, what children at-risk have in common relative to their typically developing peers is weaknesses in rapid retrieval of information, be it in the form of sound-letter correspondences (e.g., Erbeli, Hart, Wagner, & Taylor, 2018) or addition facts (e.g., Hanich, Jordan, Kaplan, & Dick, 2001). As such, it is less probable to expect large magnitudes in math paths predicting subsequent reading in academically at-risk children since the underlying ability of retrieval to facilitate such contribution is compromised in this population.

Lastly, given that the sequence of stages in reading and math development follows similar patterns (note reading and math developmental theories; Ehri, 2008; Geary, 1994; Perfetti, 1995) in all children, it is plausible to suspect that reading and math might show bidirectional influences also in academically at-risk children.

Taken together, similarly to research on typically developing children, developmental dynamics between reading and math in children who might be academically at-risk is incorporated in conventional theories showing that cognitive, reading, and math components work interdependently also in this population. Theoretically relevant pathways have been supported by empirical models indicating overlap and prospective predictions between the constructs. However, the mixed findings regarding the associations reflect the broader literature that has been inconclusive about the nature of these influences. Therefore, the current study addresses this inconclusiveness of prior work and clarifies the co-developmental pathways between reading and math in academically at-risk elementary school children.

1.3 | Modeling the developmental dynamics between reading and math

Specific methods are required to delineate the co-developmental pathways. Newer methodological approaches, such as the latent change score model, can assist in elucidating the reading-math dynamics (Ferrer & McArdle, 2010; McArdle, 2009). The model has been successfully applied in the reading (e.g., Quinn, Wagner, Petscher, & Lopez, 2015) as well as math literature (e.g., Gilbert & Fuchs, 2017) to study the co-development of only reading related or math related components. The bivariate latent change score model simultaneously models three types of change across two constructs: a constant change for each construct (i.e., linear growth), a proportional change for each construct (i.e., time-point to time-point change), and coupling effects between the constructs (i.e., development in one construct influences subsequent development in the other construct). By using this model, we are able to determine the extent to which the relation between reading and math is purely correlational in nature (i.e., correlated constant change parameters and no temporal coupling effects), or described by either unidirectional coupling effects (i.e., reading-to-math coupling model or math-to-reading coupling model) or bidirectional coupling effects (i.e., coupling effects in both directions). The crucial feature of this model will help us understand the degree to which resulting unidirectional or bidirectional coupling effects lead to increasing or decreasing rates of change in either one or both constructs (e.g. reading performance might predict the change in growth in math performance).

1.4 | The present study

The purpose of the present study was to further investigate the tenability of the abovementioned developmental relations between reading and math in an academically at-risk sample from Texas spanning elementary grades. The focus was on first through fourth grades as it is during this developmental period when formal reading and math instruction heavily influences academic achievement outcomes. Substantial progress occurs on the transition from learning to read to reading to learn (Chall, 1983) and also on multi-digit calculations (Fuchs, Geary, Fuchs, Compton, & Hamlett, 2014) during elementary grades. Importantly, since co-developmental dynamics between reading and math can be, in part, attributable to general cognitive abilities (e.g., Cattell, 1987; Pennington, 2006), in an additional model we controlled for children's nonverbal cognitive abilities to examine the dynamics beyond the influence of this underlying cognitive factor.

In the current study we utilized a latent change score model to elucidate and directly investigate potential directions of co-development effects of reading and math. This is a significant improvement from previous research as it has allowed for simultaneous estimation of growth and cross-lagged associations across different levels of reading and math achievement. Understanding developmental processes that may strengthen or weaken the longitudinal association between reading and math has implications for which children (i.e. those with high or low levels of reading or math performance) would most benefit from prevention efforts and specific interventions.

Based on the fact that all directions of pathways have received support in prior theoretical as well as empirical work, the present study offers no a priori expectation regarding which direction of influence, per se, explains the reading-math developmental dynamics most parsimoniously. However, we hypothesized for each individual possibility its underlying nature of the association. (a) If reading and math are simply correlated with no direct time-dependent relations, then based on Pennington's model (2006), we predicted reading and math initial performance and growth to be positively correlated. (b) If reading influences subsequent math growth, then in accordance with math theories (Dehaene, 1992; LeFevre et al., 2010) and based on empirical studies (Grimm, 2008; Jordan et al., 2003), we hypothesized higher levels of reading performance to produce direct positive effects on subsequent math growth. (c) If math influences subsequent reading growth, then we expected smaller magnitudes of effects compared to models in which reading influences math. (d) If reading and math co-develop reciprocally, then based on developmental theories (e.g., Ehri, 2008; Geary, 1994) positing similarities in reliance on procedural strategies in reading and math development, we anticipated bidirectional influences. To the extent that reciprocal effects might be present, we expected the effects to be stronger from reading to subsequent math growth than vice versa.

2 | METHOD

2.1 | Participants

Participants for this study were children enrolled in a 12-year prospective longitudinal study Project Achieve, which began in the fall of 2000 or 2001 when children were in first grade (Hill & Hughes, 2007; Hughes, 2015). The project examined the effects of retention in elementary grades on children's future academic achievement and socio-emotional adjustment. Eligible participants were selected based on scoring below the median on a district-administered Texas Essential Knowledge and Skills early literacy test (Texas Education Agency, 2004) in the spring of kindergarten or fall of first grade. As such, they might be classified as academically at-risk children. Exclusion criteria were: receiving special education services in first grade (if children later received special education services they were not removed from the study), already having been retained in first grade, and first language any other than English or Spanish. At the time of subject recruitment, parents were not explicitly asked if their children had neurodevelopmental disorders. A total of 1,374 children were eligible to participate, of which 784 (56.3%) consented to be enrolled in the study. There were no significant differences on age, gender, ethnicity, bilingual class placement, eligibility for free or reduced lunch, or literacy test scores between children with and without consent to participate in the study (see Hill & Hughes, 2007, for more details).

Data reported from the current study were obtained from 554 children. To avoid differences between children stemming from different grade-level exposure, we excluded 230 out of 784 children from the analyses as these children were retained either in grade 2 (N = 165, 21%), grade 3 (N = 36, 4.6%), or grade 4 (N = 29, 3.7%). There were no significant differences on sex, ethnicity, parental education level, and literacy scores between the excluded children and the rest of the sample. Importantly, the sample was not discrepant in terms of frequency of children who were below one standard deviation in reading in first grade, but had average math scores (N = 47) or vice versa (N = 48). During the first assessment time point, children were on average 6½ years old (M = 6.57; SD = 0.38; range = 5.25–8.25). Participants were enrolled from a total of 34 schools across three districts in the state of Texas. The demographic characteristics of the sample were the following: 18.9% of the children were African American, 39.8% Latino, 35.5% White, and 5.8% were classified as Other. Regarding the socio-economic status of participants, 56.8% were eligible for free or reduced lunch.

2.2 | Procedure and measures

Detailed information on measure administration and procedures is provided in Supplementary Materials.

2.2.1 | Reading

Reading was measured using the Woodcock-Johnson Test of Achievement – 3rd ed. (WJ–III) Broad Reading measure (WJ-Reading; Woodcock et al., 2001). A composite WJ-Reading score was utilized. It included letter-word identification, reading fluency, and passage comprehension. *W*-scores were used here as they are ideal for modeling growth. The *W*-score is centered on a value of 500, which is set to approximate the average performance of a typical child at age 10 (grade 5). The reliability and construct validity of the WJ-Reading have been extensively documented (Woodcock & Johnson, 1990; Woodcock et al., 2001). The published internal consistency reliability is 0.92 (ages 4–7) and 0.93 (ages 8–10; Woodcock et al., 2001). No raw (item-level) data for reading were available to the researchers, so calculating sample specific reliability was not possible.

2.2.2 | Math

Math was measured using the WJ–III Broad Math measure (WJ-Math; Woodcock et al., 2001). A composite WJ-Math score was used. It consisted of calculations, math fluency, and applied problems. Similarly to reading, we used *W*-scores. Extensive research documents the reliability and construct validity of the WJ-Math (Woodcock & Johnson, 1990; Woodcock et al., 2001). The published internal consistency reliability estimates are 0.94 (ages 4–7) and 0.92 (ages 8–10; Woodcock et al., 2001). No raw (item-level) data for math were available to the researchers, so calculating sample specific reliability was not possible.

The Batería Woodcock–Muñoz: Pruebas de Aprovechamiento–Revisada (Batería–R; Woodcock & Muñoz-Sandoval, 1996) is the comparable Spanish version of the Woodcock-Johnson Psycho-Educational Battery–Revised (WJ–R; Woodcock & Johnson, 1990). The published internal consistency reliability estimates for both reading and math are 0.95 (Woodcock & Muñoz-Sandoval, 1996). If children or their parents spoke any Spanish, children were administered the Woodcock–Muñoz Language Survey (Woodcock & Muñoz-Sandoval, 1993) to determine the child's language proficiency in English and Spanish and selection of either the WJ–III or the Batería–R. The WJ–III or the Bateria–R were then administered in the language in which the student demonstrated greater language proficiency on the Woodcock–Muñoz Language Survey. If the student demonstrated equal or greater language proficiency in English for three consecutive years, subsequent WJ–III tests were administered in English. The Woodcock Compuscore program (Woodcock & Muñoz-Sandoval, 2001) yields *W*-scores for the Batería–R that are comparable to *W*-scores on the WJ–R. In our study, 13.9% of children included in the analyses were administered the Spanish version of tests. Posthoc analyses showed that for all four grades, children tested in Spanish had significantly higher scores in reading and lower scores in math compared to the children tested in English. Importantly, results of the main analyses were replicated even when we excluded children tested in Spanish from the sample. Therefore, we report the results using the full sample.

2.2.3 | Nonverbal cognitive abilities

Nonverbal cognitive abilities were measured using the Universal Nonverbal Intelligence Test (UNIT; Bracken & McCallum, 1998). The UNIT is a nationally standardized nonverbal measurement of general intelligence and cognitive abilities. It measures complex memory and reasoning abilities. The UNIT has demonstrated good internal consistency reliability (0.84) as well as construct validity (Bracken & McCallum, 1998).

2.3 | Data analyses

2.3.1 | Descriptive statistics and correlational analyses

Descriptive statistics among all observed variables using *W*-scores were calculated in SAS 9.4. Age effect can bias results, hence raw data on reading and math in first grade were corrected by residualizing on age. Residualized data were subsequently standardized to scaled *z*-scores (see Supplementary Materials section). Correlations were calculated on residualized and standardized data. All subsequent analyses were also conducted on residualized and standardized data.

2.3.2 | Latent change score modeling

To investigate the developmental dynamics between reading and math over time, we conducted latent change score analyses (McArdle, 2009). For more detailed descriptions of data analyses, please refer to the Supplementary Materials section.

3 | RESULTS

3.1 | Descriptive statistics and correlational analyses

Descriptive statistics are provided in Table 1. There was an expected pattern of increasing performance across elementary grades on both

TABLE 1 Descriptive statistics

Observed variables	*N*	*M* (*SD*)	Skew	Min	Max
1. Reading grade 1	540	439.16 (26.21)	0.41	375.00	523.00
2. Reading grade 2	495	468.24 (18.68)	−0.35	406.00	521.00
3. Reading grade 3	478	482.47 (17.02)	−0.83	394.00	548.00
4. Reading grade 4	472	492.90 (17.08)	−0.52	412.00	565.00
5. Math grade 1	541	464.17 (13.45)	−1.08	384.00	498.00
6. Math grade 2	495	478.15 (10.14)	−0.59	438.00	502.00
7. Math grade 3	477	489.60 (10.12)	−0.74	448.00	517.00
8. Math grade 4	471	499.32 (9.49)	−0.51	464.00	530.00
9. Nonverbal cognitive abilities grade 1	547	94.45 (14.75)	−0.35	48.00	132.00

Abbreviations: *M* = mean; Max = maximum; Min = minimum; *N* = number of children; *SD* = standard deviation; Skew = skewness.

constructs. This is also depicted in Figure 1 with observed trajectories for both constructs plotted over time. Both constructs demonstrated relatively stable variability over time. The average nonverbal cognitive abilities score was slightly below 100 but still within the average intelligence range. Correlations residualized on age are presented in Table 2. Correlational magnitudes between measures within the same academic construct were moderate to relatively high, whereas those across constructs were moderate. Academic outcomes were only weakly correlated with nonverbal cognitive abilities.

3.2 | Latent change score modeling

3.2.1 | Univariate latent change score models

A series of competing univariate models were fit to the data residualized on age separately for reading and math. Model-fitting results for the set of univariate reading and math models are summarized in Table 3. As shown in Table 3, the constant change models (models R1 and M1, respectively) and the proportional change models (models R2 and M2, respectively) fit poorly for both constructs. However, the dual change models for reading as well as math (models R3 and M3, respectively) fit data well. Removing the constant change portions from the dual change models resulted in significantly worse model fit in both constructs, indicating that the constant change paths needed to be retained in the models. Similarly, removing the proportional change portions of the models from the dual change models also resulted in a severe degradation of model fit for both constructs. Thus, the dual change models for reading and math were retained for bivariate latent change score analyses. Overall, the univariate models indicated that adequate modeling of reading and math development required both constant change and proportional change parameters.

Unstandardized parameter estimates from the best fitting univariate dual change score model for reading are presented in Figure 2. The mean intercept, or average initial scores in first grade, was not significantly different from zero (μ_{0R} = 0.027), which was expected given the conversion of *W*-scores to scaled *z*-scores. There was significant variability in the initial means, alluding to individual differences in the baseline values (σ_{0R} = 0.596). There was a significant growth rate in reading (μ_{1R} = 1.049), with significant individual differences across children (σ_{1R} = 0.063). The proportional change parameter was significant and negative (β_R = −0.406), indicating that children who performed higher at a certain grade showed smaller change in growth in the subsequent grade. The positive correlation between the intercept and slope ($r_{R0,1}$ = 0.644) indicated a fan-spread growth pattern, where, on average, children who were below the mean in the first grade grew more slowly across the span of first through fourth grade than children who were above the mean.

Figure 3 presents unstandardized parameter estimates for the dual change score model for math, which fit best to our data. The results were similar to reading parameters in terms of directionality. The mean intercept was not statistically different from zero (μ_{0M} = −0.007). There were significant individual differences in the level of performance in first grade (σ_{0M} = 0.624). A positive slope in math was significant (μ_{1M} = 1.043), indicating that in the absence of other influences, there was a tendency for a positive growth rate, and this estimate varied significantly across individuals (σ_{1M} = 0.033). Similar to reading, the proportional change parameter was significant and negative (β_M = −0.177), indicating an overall attenuated growth in math over time. In other words, children who performed higher at a certain grade showed smaller change in growth in the subsequent grade compared to children who performed lower. The correlation between the intercept and slope was positive ($r_{M0,1}$ = 0.336), suggesting that children below the mean grew on average more slowly than children above the mean.

In sum, given that univariate dual change score models for both constructs provided the best fit for our data, we proceeded with a bivariate latent change score modeling to study potential coupling effects between reading and math.

3.2.2 | Bivariate dual change score models

Bivariate dual change score analyses were conducted for four alternative models: a correlated, but uncoupled model, a reading-to-math coupling model, a math-to-reading coupling model, and a full, bidirectional coupling model. The critical parameter in the bivariate dual change score models was the coupling-effects parameter. Covariances were also estimated between reading and math intercepts and slopes.

A summary of the bivariate model-fitting is presented in Table 3. The model-fitting results indicated that all four bivariate models fit the data well. Removing both coupling effects (model RM1) from the full, bidirectional model (model RM4) resulted in a significant deterioration of fit, indicating that one or both coupling paths were

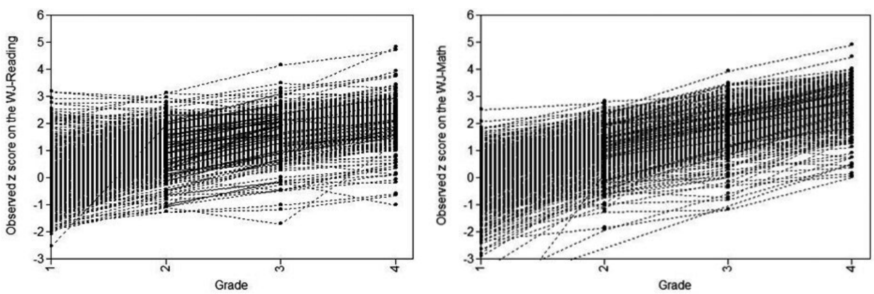

FIGURE 1 Observed trajectories for the WJ-Reading and WJ-Math. All values were converted to *z*-scores based on means and standard deviations in grade 1

TABLE 2 Correlations among observed variables

Observed variables	1	2	3	4	5	6	7	8
1. Reading grade 1	–							
2. Reading grade 2	0.65**	–						
3. Reading grade 3	0.59**	0.82**	–					
4. Reading grade 4	0.54**	0.79**	0.86**	–				
5. Math grade 1	0.30**	0.15**	0.21**	0.25**	–			
6. Math grade 2	0.13**	0.23**	0.28**	0.32**	0.68**	–		
7. Math grade 3	0.26**	0.30**	0.41**	0.41**	0.63**	0.70**	–	
8. Math grade 4	0.35**	0.39**	0.48**	0.51**	0.55**	0.65**	0.78**	–
9. Nonverbal cognitive abilities grade 1	0.20**	0.18**	0.20**	0.23**	0.37**	0.24**	0.27**	0.27**

Note: Correlations were calculated on data residualized on age.
**$p < .0001$.

TABLE 3 Summary of univariate and bivariate model fitting and model comparisons

		Model fit χ^2 (*df*)	RMSEA (95% CI)	CFI	SRMR	Model comparisons $\Delta\chi^2$ (Δ*df*)	*p*
Univariate models							
Reading	R1 Constant change model	297.442 (5)	0.326 (0.295, 0.358)	0.795	0.154	R1 versus R3 283.032 (1)	<.0001
	R2 Proportional change model	786.048 (7)	0.449 (0.423, 0.476)	0.455	0.351	R2 versus R3 771.638 (3)	<.0001
	R3 Dual change model	**14.410 (4)**	**0.069 (0.033, 0.109)**	**0.993**	**0.016**	N/A	N/A
Math	M1 Constant change model	87.865 (5)	0.173 (0.143, 0.206)	0.926	0.070	M1 versus M3 76.421 (1)	<.0001
	M2 Proportional change model	750.444 (7)	0.439 (0.413, 0.466)	0.335	0.341	M2 versus M3 739.000 (3)	<.0001
	M3 Dual change model	**11.444 (4)**	**0.058 (0.020, 0.099)**	**0.993**	**0.042**	N/A	N/A
Bivariate models							
Reading – math	RM1 Correlated, but uncoupled development	99.300 (19)	0.088 (0.071, 0.105)	0.971	0.042	RM1 versus RM4 10.500 (2)	<.01
	RM2 Reading-to-math coupling model	**89.872 (18)**	**0.085 (0.068, 0.103)**	**0.974**	**0.036**	**RM2 versus RM4 1.072 (1)**	**.300**
	RM3 Math-to-reading coupling model	98.379 (18)	0.090 (0.073, 0.108)	0.971	0.042	RM3 versus RM4 9.579 (1)	<.01
	RM4 Full, bidirectional coupling model	88.800 (17)	0.088 (0.070, 0.106)	0.974	0.035	N/A	N/A

Note: Model fitting was conducted on data residualized on age. Selected best fitting models are in bold type.
Abbreviations: CFI = comparative fit index; *M* = math; R = reading; RM = reading-math; RMSEA = root mean squared error of approximation; SRMR = standardized root mean square residual.

significant. When we removed the reading-to-math coupling effect (model RM3) from the full, bidirectional model, model comparison revealed a significant deterioration of fit. In contrast, removing the math-to-reading coupling effect (model RM2) from the full model did not result in model fit deterioration. In summary, this means that results from the chi-square difference testing supported the bivariate latent change score model with reading-to-math coupling effects (model RM2) as the most parsimonious and well-fitting model that best described our data.

Unstandardized parameter estimates of the most parsimonious model are presented in Figure 4. The positive correlations between the intercept and slope for reading ($r_{R0,1}$ = 0.619) and math ($r_{M0,1}$ = 0.347) reflected the fan-spread growth model, mirroring the findings in the univariate dual change score models. The correlations between reading intercept and math slope ($r_{R0,M1}$ = 0.098), between math intercept and reading slope ($r_{M0,R1}$ = 0.268), and between reading intercept and math intercept ($r_{R0,M0}$ = 0.242) alluded to the fact

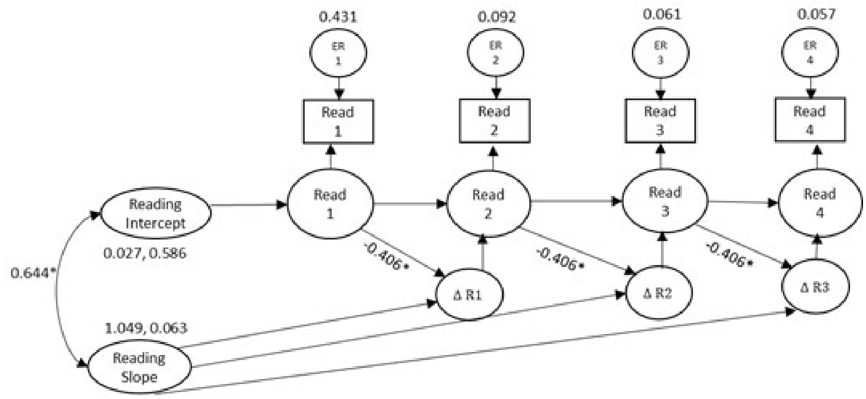

FIGURE 2 Univariate dual change score model for reading. Model fitting was conducted on data residualized on age. Statistics are unstandardized path coefficients. Paths with no coefficients were constrained to 1. ER, error variance for observed variables in reading; R, reading. *$p < .001$

that initial levels and growth in reading and math were positively correlated.

As shown in Figure 4, significant coupling effects ($\gamma = 0.165$) were depicted from the level of performance in reading to subsequent change in math growth. This effect indicated that annual change in math growth was accounted for, in part, by level of performance in reading. Specifically, a child whose level of reading was one standard deviation above that of an average child would grow 0.165 standard deviations faster in math over a year. This result indicates that reading performance might act as an amplifying force on math growth.

As for the residual variances, we can see in Figure 4 that they dropped over the elementary grades. The largest variances observed at the first time point likely reflected individual differences before children started formal schooling in elementary grades. The transition from kindergarten to first grade might have influenced the differences, such that they decreased and became more constant across children as they progressed through schooling.

One way to illustrate dynamic relation between reading and math is by drawing a statistical vector plot (Ferrer & McArdle, 2004), which represents the projections in time (i.e., grade changes) for different combinations for pairs of constructs. The plot, as depicted in Figure 5, is based on the expected values from the best fitting model. Even though the best fitting model has a fixed set of group parameters, the expected gains change as a function of the current status of any individual. In this vector field an arrow is plotted as the direction of change expected for any given pair of scores in the *X-Y* plane (Ferrer & McArdle, 2004). As such, the tail of the arrow represents a specific starting point for reading and math, and the head of the arrow represents the expected values of reading and math at the next occasion (i.e. next grade, which equals to 1 year). The length of the arrow indicates the size of the expected change. Figure 5 includes at least three noteworthy characteristics. First, the coupling effects from reading to math are evident in particular at the mean and high levels of reading performance (reading $z > 0$) and below the mean levels of math performance (math $z < 0$). Average and high reading performance levels appeared to have their strongest amplifying effects (the steepest arrows) over time on math growth for below average math performing children, as indicated on the bottom right side of the vector field. We can see reasonable expected change (long arrows) in math growth for these children. Second, and conversely, weakest amplifying effects (flatter or almost horizontal arrows) of reading on math growth were on the left side of the vector plot (reading $z < -1$). At the lowest levels of reading performance, there was a negligible or no expected growth in math. Third, the equilibrium of this dynamic system seems to be situated around the average score for reading (reading $z = 0$) and the above average score for math (math $z > 1$). For those children, the dynamics were stable, without clear coupling influences from reading to math. As such, we are unable to predict reliably change in math growth from one grade to the other for those children.

After conducting a bivariate latent change score model, we added to the intercept and slope parameters a covariate representing nonverbal cognitive abilities. The fit statistics for the models with a covariate are presented in Table S1. The model-fitting results

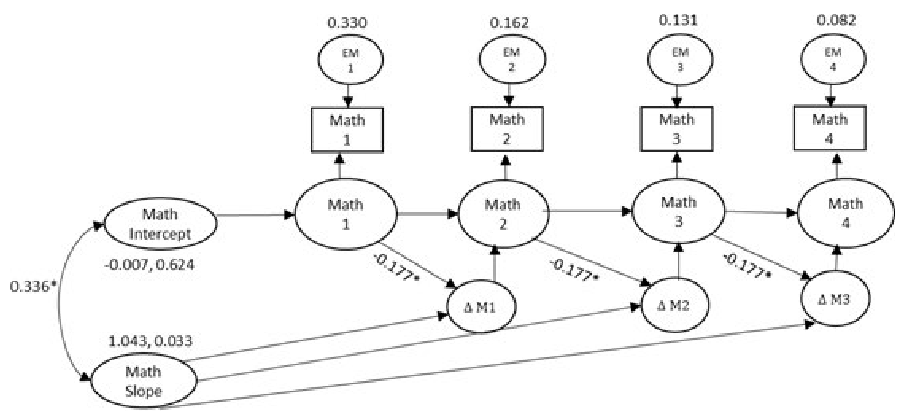

FIGURE 3 Univariate dual change score model for math. Model fitting was conducted on data residualized on age. Statistics are unstandardized path coefficients. Paths with no coefficients were constrained to 1. EM, error variance for observed variables in math; *M*, math. *$p < .001$

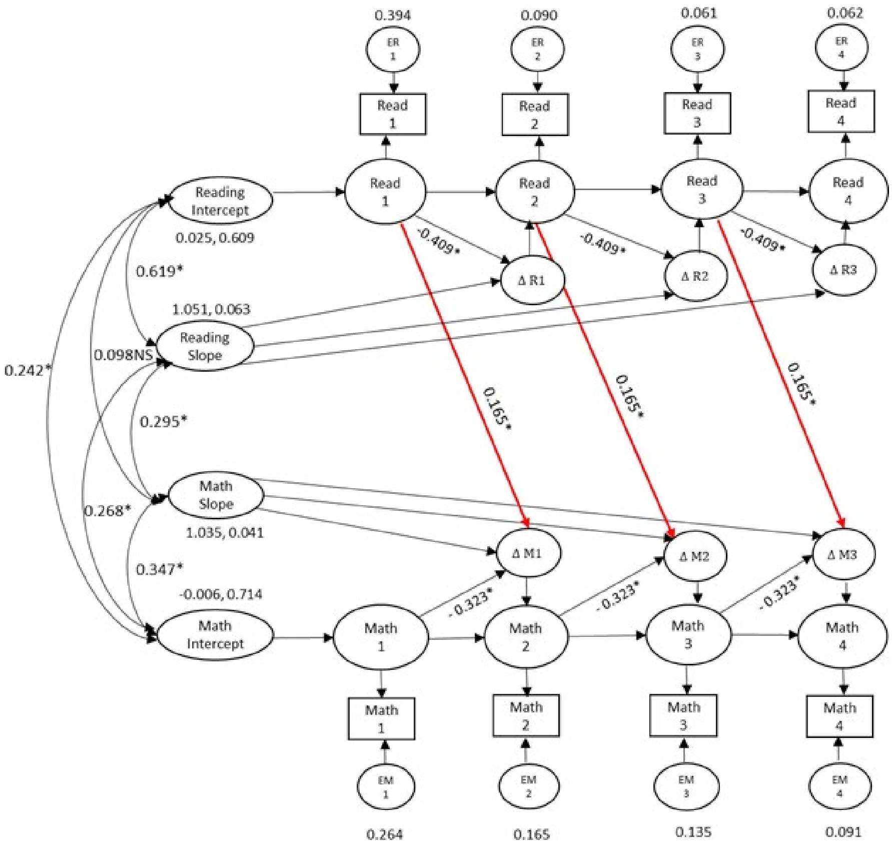

FIGURE 4 Best fitting bivariate latent change score model with coupling effects running from reading to math. Model fitting was conducted on data residualized on age. Statistics are unstandardized path coefficients. Paths with no coefficients were constrained to 1. EM, error variance for observed variables in math; ER, error variance for observed variables in reading; *M*, math; NS, non-significant; R, reading. **p* < .001

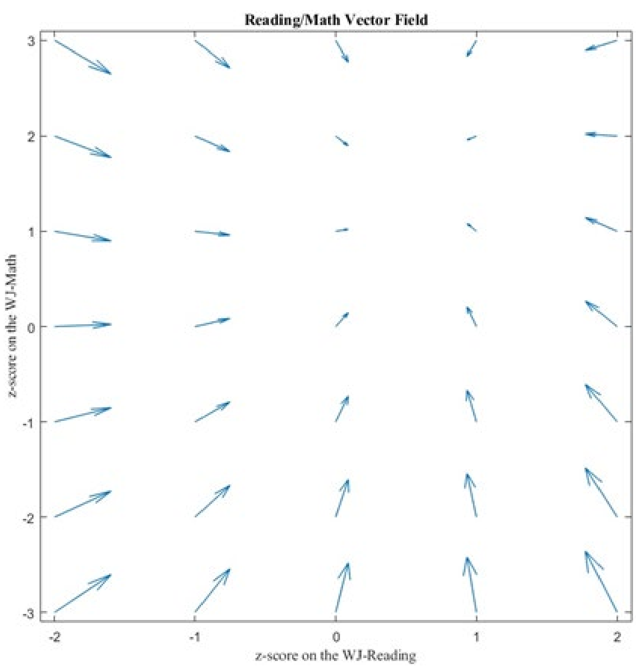

FIGURE 5 Statistical vector field plot for the reading-math bivariate dynamics system from the best-fitting model. Arrow directions and arrow lengths indicate the expected direction and magnitude of change in both constructs after a subsequent time period (in our case, this represents next grade, which equals to 1 year)

with nonverbal cognitive abilities as a covariate supported the same best fitting model without the covariate, indicating that the findings regarding reading influencing subsequent math growth as the most parsimonious model held. As indicated in Figure S1, the path estimates changed minimally and nonverbal cognitive abilities were significantly related to all intercept and growth parameters.

4 | DISCUSSION

The life-long influence of acquiring adequate reading and math skills on children's lives demonstrates the importance to understand their developmental dynamics. Given the differences between these two constructs, yet the great extent of their overlap, a fundamental question is whether and to what degree both skills unfold in conjunction over time. The aim of this study was to explore developmental dynamics between reading and math in academically at-risk children across elementary schooling. Overall, our results suggested that for these children, reading performance was significantly associated with increased change in math growth. Specifically, relatively average and high levels of reading performance were associated with subsequent gains in math growth, in particular for children performing below average in math. In contrast, relatively low levels of reading performance had negligible or no amplifying influences on change in math growth. Our findings were robust even after controlling for nonverbal cognitive abilities.

The univariate latent change score models suggested complex trajectory lines for both constructs. Longitudinal trajectory was best represented using models involving constant and proportional elements. These models revealed that the changes were similar in nature for reading and math, showing a slowing of nonlinear progress, whereby reading indicated larger decreases in change throughout elementary school compared to math. Finding larger decreases in change in reading than math was not consistent with our expectation because, traditionally, more emphasis is placed on reading than math in U.S. elementary schools (Lee & Reeves, 2012). The decelerated effects also revealed that much of the learning occurs within the early phases of learning to read and doing math.

Turning to the bivariate model examining the dynamic development between both constructs, we tested four scenarios based on the theoretical background. The results supported the reading-to-math direction and suggested that reading appeared to be a leading and math a lagging indicator of this dynamics over time, even when we held nonverbal cognitive abilities constant. As hypothesized, the effect was positive, yet only small in magnitude. Under a simple view and in line with Grimm (2008), this would mean that children applied their knowledge of reading to their math skills. From this perspective, patterns of change in math were dependent on reading-math combinations, rather than on the math construct alone. The unidirectional reading to subsequent math finding presents a point of departure from the results of studies conducted on national datasets which showed bidirectional influences (e.g., Bailey et al., 2020). The simplest explanation for the difference is that those studies examined children across the performance continuum, whereas the present study examined children who might have been academically at-risk. However, even nationally representative studies showed that the cross-domain estimates from math to subsequent reading were small in magnitude – mostly between 0 and 0.1 (e.g., Bailey et al., 2020). Nonetheless, the inconsistency in the directionality of influences across populations warrants further attention.

The results of this study directly point to the premise that it makes little sense to talk about the extent to which reading and math are associated without also considering a child's current level of reading achievement. Why does overall achievement status in reading play a positive significant role in the prediction of changes in growth rates in math in academically at-risk children? There are at least two possible explanations for such cross-construct pattern. The first explanation is related to the significant correlated cross-construct intercept and slope parameters. Reading and math rely on the same domain-general factors. Even though reading and math might not be reversible processes according to our results, they seem to share overarching cognitive domains, which in conjunction contribute to the relation between them. We controlled for one cognitive factor, nonverbal ability, however, in accordance with the multiple deficit model (Pennington, 2006), there might be other correlated cognitive risk liabilities that are shared across achievement outcomes in academically at-risk children. For example, processing and working memory deficits have been shown to contribute to reading-math pairwise overlap (e.g., Daucourt et al., 2020; Moll, Gobel, Gooch, Landerl, & Snowling, 2016; Swanson et al., 2009; Willcutt et al., 2013).

The second mechanism pertains to the role of shared reliance on linguistic elements across constructs. As noted, mathematical developmental theories (e.g., Dehaene, 1992; LeFevre et al., 2010) have supported the role of language and phonological awareness in the development of adequate math achievement through, for example, successful linking of visual-verbal associations of number codes. Moreover, it is even assumed that math facts are stored as phonological codes in long-term memory (Simmons & Singleton, 2008), again supporting the crucial role of phonological system for early math outcomes. Many empirical studies have, in fact, corroborated the pivotal role of reading related skills for math prediction in children with learning disabilities (e.g., Jordan et al., 2003; Joyner & Wagner, 2019; Swanson & Jerman, 2006; Swanson et al., 2009). Taken together, it appears that in academically at-risk children, the ability to read well may have facilitated changes in the math growth rate through linguistic elements that had been acquired via the processing of the phonological code of the language.

As a whole, the dynamic system between reading and math pointed toward improvement in changes in math growth via reading performance at the next occasion of measurement (i.e. next grade). More specifically, though, changes in math growth rates from one grade to the next were determined, in part, by the level of reading achievement in that grade. As hypothesized, the amplifying influence of reading performance on math growth gains was evident at relatively average and high levels of reading and relatively below average of math performance, suggesting that high reading skills served to increase changes in math growth, in particular for children who had math difficulties. This finding is in line with Grimm (2008) and Jordan et al. (2003) which showed that higher levels of reading performance influenced math growth in typically developing children as well as children with math disabilities. This finding alludes to reading and math having, in part, synergistic effects that drive the dynamics between both constructs toward a picture of improved performance in academically at-risk children. It appears that well-established long-term phonological representations without underlying phonological retrieval deficits (both skills are a hallmark of good readers) guided subsequent growth in math. Previous research has shown that it is particularly the quality of long-term phonological representations that is important for specific arithmetic skills, namely those that have a higher probability of being solved by retrieval (De Smedt, Taylor, Archibald, & Ansari, 2010).

A contrasting picture, on the other hand, could be observed at low levels of reading performance. Poor reading performance dampened changes in math growth. Math growth reduction following decreases in reading performance levels is compatible with the growing evidence showing that the presence of reading disability is associated with greater risk for math disability (e.g. see the meta-analysis by Joyner & Wagner, 2019). The lagged nature of the analyses indicated that a low performance level in reading increased the probability of subsequent little or no change in the growth rate of math. All in all, such a finding informs educational literature more broadly in that it helps us understand secondary math deficits by demonstrating that math problems might, at the same time, be related to reading problems.

Our study contributes some imperative educational implications. The study shows that reading developmental processes strengthed the longitudinal association between reading and math, in particular, for children with higher levels of reading performance and lower levels of math performance. This finding implies that it would be worthwhile to implement interventions, in particular ones targeting language and reading for children who exhibit lower levels of reading and are at-risk for math difficulties (e.g., Fuchs et al., 2020). Tracking the subsequent results in both reading and math performance would be particularly valuable as the results would show us whether immediate impact on language and reading would be expected to also boost children's math performance. We note that the cross-construct effect in our study might be small in magnitude (γ = 0.165). Nonetheless, it would be interesting to expand current interventions to effectively consider both reading and math outcomes. Importantly, we do not call for one-way evidence of experimental interventions aimed at targeting only reading to work to remediate math deficits. We do not exclude the possibility that interventions targeting math deficits might transfer and help ameliorate reading difficulties. In fact, there is evidence from intervention research showing that remediating math also indicated gains on reading skills when solving word problems (e.g., Fuchs et al., 2008).

The present results should be considered in light of some limitations and directions for future research. First, the representativeness of the results is limited to our academically at-risk sample. Stringent criteria for participant selection were used in the current study. However, our sample selection may also be viewed as a strength for expanding similar existing work which has mostly been based on typically developing children. Nonetheless, it remains possible that the dynamics of our constructs unfold differently in other populations, including children with learning disabilities. This might be a direction for future research. Second, we examined grade-to-grade changes across a span of elementary school in reading and math broadly defined. Future research could aim at modeling subtler shifts occurring over briefer time periods in various reading and math components. For instance, the exploration of more proximal relations between reading fluency and math fluency following instruction in decoding could be a direction for future investigation. Third, our study was not designed to pinpoint what specific overlapping cognitive factors may explain the reading-math dynamics. These factors have been, in part, explored in other important work (e.g., Cirino, Childs, & Macdonald, 2018; Koponen et al., 2019; Korpipää et al., 2019). We controlled for one such factor, nonverbal cognitive abilities. Nonetheless, it would be helpful to be able to incorporate at least some other cognitive correlates, such as processing speed and working memory. Unfortunately, these data were not collected for our sample.

5 | CONCLUSION

Elementary school is one of the most crucial developmental periods for a child's academic success in reading and math. This study sheds valuable light on the degree to which reading and math unfold in conjunction over the course of elementary school in academically at-risk children. Findings illustrate that good reading skills pave the way for at-risk children to develop their math skills, in particular if they perform below average on math. Understanding the impacts of reading on math requires an appreciation of co-developmental influences, which has implications for early instruction and intervention.

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CONFLICT OF INTEREST

None declared.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section.

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